Rectangle Theorem

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Abstract

I claim that every rectangle with sides an_a, bn_b and with diagonal(s) dn_d is a transformation of a rectangle with a side a equal to a side b, $a = b$, and with diagonal(s) d which corresponds to a rectangle with a side a_1 equal to 1, $a_1 = 1$, side b_1 equal to 1, $b_1 = 1$, and with diagonal(s) d_1 equal to $\sqrt{2}$, $d_1 = \sqrt{2}$, corresponding transformation of which results in a transformation of the rectangle with the side a_1 equal to 1, $a_1 = 1$, side b_1 equal to 1, $b_1 = 1$, and with diagonal(s) d_1 equal to $\sqrt{2}$ into a rectangle with sides $a_1 n_{a_1}$, $b_1 n_{b_1}$ and with diagonal(s) $d_1 n_{d_1}$ which being multiplied by the side $a = b$ of the rectangle with the side a equal to the side b, $a = b$, and with diagonal(s) d accordingly produce the rectangle with sides an_a, bn_b and diagonal(s) dn_d and I prove that this what I claim is true.

Contents

1 Introduction

I claim that every rectangle with sides an_a, bn_b and with diagonal(s) dn_d is a transformation of a rectangle with a side a equal to a side b, $a = b$, and with diagonal(s) d which corresponds to a rectangle with a side a_1 equal to 1, $a_1 = 1$, side b_1 equal to 1, $b_1 = 1$, and with diagonal(s) d_1 equal to $\sqrt{2}$, $d_1 = \sqrt{2}$, corresponding transformation of which results in a transformation of the rectangle with the side a_1 equal to 1, $a_1 = 1$, side b_1 equal to 1, $b_1 = 1$, and with diagonal(s) d_1 equal to $\sqrt{2}$ into a rectangle with sides $a_1 n_{a_1}$, $b_1 n_{b_1}$ and with diagonal(s) $d_1 n_{d_1}$ which being multiplied by the side $a = b$ of the rectangle with the side a equal to the side b, $a = b$, and with diagonal(s) d accordingly produce the rectangle with sides an_a, bn_b and diagonal(s) dn_d and I prove that this what I claim is true as it is consistent with results given by formulas including formula of Pythagorean theorem.

2 Rectangle Theorem

Rectangle Theorem. *Every rectangle with sides an_a, bn_b and with diagonal(s) dn_d is a transformation of a rectangle with a side a equal to a side b, $a = b$, and with diagonal(s) d, the side a of which is equal to a product of the side a equal to the side b, $a = b$, of a rectangle with the side a equal to the side b, $a = b$, and with diagonal(s) d and a side a_1 of a rectangle with the side a_1 equal to a side b_1 equal to 1, $a_1 = b_1 = 1$, and with diagonal(s) d_1 equal to $\sqrt{2}$, $d = \sqrt{2}$:*

$$a = aa_1 \tag{1}$$

or

$$a = ba_1 \tag{2}$$

the side b of which is equal to a product of the side a equal to the side b, $a = b$, of a rectangle with the side a equal to the side b, $a = b$, and with diagonal(s) d and a side b_1 of the rectangle with the side a_1 equal to the side b_1 equal to 1, $a_1 = b_1 = 1$, and with diagonal(s) d_1 equal to $\sqrt{2}$, $d = \sqrt{2}$:

$$b = ab_1 \tag{3}$$

or

$$b = bb_1 \tag{4}$$

the diagonal(s) d of which is(are) equal to a product of the side a equal to the side b, $a = b$, of a rectangle with the side a equal to the side b, $a = b$, and with diagonal(s) d and a diagonal d_1 of the rectangle with the side a_1 equal to the side b_1 equal to 1, $a_1 = b_1 = 1$, and with diagonal(s) d_1 equal to $\sqrt{2}$, $d = \sqrt{2}$:

$$d = ad_1 \tag{5}$$

or

$$d = bd_1 \tag{6}$$

by the change of its side or sides $a = b$ and diagonal(s) d accordingly by accordingly the same corresponding rates characterizing relations between the transformed sides an_a, bn_b and diagonal(s) dn_d of the rectangle with the sides an_a, bn_b and with diagonal(s) dn_d and the side a equal to the side b, $a = b$, of the rectangle with the side a equal to side b, $a = b$, and with diagonal(s) d as rates characterizing relations between the transformed sides $a_1n_{a_1}$, $b_1n_{b_1}$ and diagonal(s) $d_1n_{d_1}$ of the rectangle with the sides $a_1n_{a_1}$, $b_1n_{b_1}$ and with diagonal(s) $d_1n_{d_1}$ and the side a_1 equal to the side b_1, $a_1 = b_1$, of the rectangle with the side a_1 equal to the side b_1 equal to 1, $a_1 = b_1 = 1$, and with diagonal(s) d_1 equal to $\sqrt{2}$, $d = \sqrt{2}$:

$$n_a = \frac{an_a}{a} = \frac{a_1n_{a_1}}{a_1} = n_{a_1} \tag{7}$$

$$n_b = \frac{bn_b}{a} = \frac{b_1n_{b_1}}{a_1} = n_{b_1} \tag{8}$$

$$n_d = \frac{dn_d}{a} = \frac{d_1n_{d_1}}{a_1} = n_{d_1} \tag{9}$$

or accordingly:

$$n_a = \frac{an_a}{b} = \frac{a_1n_{a_1}}{b_1} = n_{a_1} \tag{10}$$

$$n_b = \frac{bn_b}{b} = \frac{b_1n_{b_1}}{b_1} = n_{b_1} \tag{11}$$

$$n_d = \frac{dn_d}{b} = \frac{d_1n_{d_1}}{b_1} = n_{d_1} \tag{12}$$

corresponding to transformatin constituting rectangle with the sides $a_1n_{a_1}$, $b_1n_{b_1}$ and with diagonal(s) $d_1n_{d_1}$ being the transformation of the rectangle *with the side* a_1 *equal to the side* b_1 *equal to 1,* $a_1 = b_1 = 1$*, and with diagonal(s)* d_1 *equal to* $\sqrt{2}$*,* $d = \sqrt{2}$ *by the change of its side or sides* $a_1 = b_1 = 1$ *and diagonal(s)* $d = \sqrt{2}$ *accordingly by accordingly the same corresponding rates characterizing relations between the transformed sides* $a_1 n_{a_1}$*,* $b_1 n_{b_1}$ *and diagonal(s)* $d_1 n_{d_1}$ *of the rectangle with the sides* $a_1 n_{a_1}$*,* $b_1 n_{b_1}$ *and with diagonal(s)* $d_1 n_{d_1}$ *and the side* a_1 *equal to the side* b_1*,* $a_1 = b_1$*, of the rectangle with the side* a_1 *equal to the side* b_1 *equal to 1,* $a_1 = b_1 = 1$*, and with diagonal(s)* d_1 *equal to* $\sqrt{2}$*,* $d = \sqrt{2}$ *as rates characterizing relations between the transformed sides* an_a*,* bn_b *and diagonal(s)* dn_d *of the rectangle with the sides* an_a*,* bn_b *and with diagonal(s)* dn_d *and the side* a *equal to the side* b*,* $a = b$*, of the rectangle with the side* a *equal to the side* b*,* $a = b$*, and with diagonal(s)* d*:*

$$n_{a_1} = \frac{a_1 n_{a_1}}{a_1} = \frac{an_a}{a} = n_a \tag{13}$$

$$n_{b_1} = \frac{b_1 n_{b_1}}{a_1} = \frac{bn_b}{a} = n_b \tag{14}$$

$$n_{d_1} = \frac{d_1 n_{d_1}}{a_1} = \frac{dn_d}{a} = n_d \tag{15}$$

or accordingly:

$$n_{a_1} = \frac{a_1 n_{a_1}}{b_1} = \frac{an_a}{b} = n_a \tag{16}$$

$$n_{b_1} = \frac{b_1 n_{b_1}}{b_1} = \frac{bn_b}{b} = n_b \tag{17}$$

$$n_{d_1} = \frac{d_1 n_{d_1}}{b_1} = \frac{dn_d}{b} = n_d \tag{18}$$

the side an_a *of which is equal to a product of the side* a *equal to the side* b*,* $a = b$*, of the rectangle with the side* a *equal to the side* b*,* $a = b$*, and with diagonal(s)* d *and the side* $a_1 n_{a_1}$ *of the rectangle with the sides* $a_1 n_{a_1}$*,* $b_1 n_{b_1}$ *and with diagonal(s)* $d_1 n_{d_1}$ *constituting transformation of the side* a_1 *of the rectangle with the side* a_1 *equal to the side* b_1 *equal to 1,* $a_1 = b_1 = 1$*, and with diagonal(s)* d_1 *equal to* $\sqrt{2}$*,* $d = \sqrt{2}$*, corresponding to transformation of the side* a *of the rectangle with the side* a *equal to the side* b*,* $a = b$*, and with diagonal(s)* d *into the side* an_a *of the rectangle with the sides* an_a*,* bn_b *and with diagonal(s)* dn_d*:*

$$an_a = aa_1 n_{a_1} \tag{19}$$

or

$$an_a = ba_1n_{a_1} \tag{20}$$

the side bn_b *of which is equal to a product of the side* a *equal to the side* b*,* $a = b$*, of the rectangle with the side* a *equal to the side* b*,* $a = b$*, and with diagonal(s)* d *and the side* $b_1n_{b_1}$ *of the rectangle with the sides* $a_1n_{a_1}$*,* $b_1n_{b_1}$ *and with diagonal(s)* $d_1n_{d_1}$ *constituting transformation of the side* b_1 *of the rectangle with the side* a_1 *equal to the side* b_1 *equal to 1,* $a_1 = b_1 = 1$*, and with diagonal(s)* d_1 *equal to* $\sqrt{2}$*,* $d = \sqrt{2}$*, corresponding to transformation of the side* b *of the rectangle with side* a *equal to the side* b*,* $a = b$*, and with diagonal(s)* d *into the side* bn_b *of the rectangle with the side* an_a*,* bn_b *and with diagonal(s)* dn_d*:*

$$bn_b = ab_1n_{b_1} \tag{21}$$

or

$$bn_b = bb_1n_{b_1} \tag{22}$$

and the diagonal(s) dn_d *of which is(are) equal to a product of the side* a *equal to the side* b*,* $a = b$*, of a rectangle with the side* a *equal to the side* b*,* $a = b$*, and with diagonal(s)* d *and diagonal(s)* $d_1n_{d_1}$ *of the rectangle with the sides* $a_1n_{a_1}$*,* $b_1n_{b_1}$ *and with diagonal(s)* $d_1n_{d_1}$ *constituting transformation of the diagonal(s)* d_1 *of the rectangle with the side* a_1 *equal to the side* b_1 *equal to 1,* $a_1 = b_1 = 1$*, and with diagonal(s)* d_1 *equal to* $\sqrt{2}$*,* $d = \sqrt{2}$*, corresponding to transformation of the diagonal(s)* d *of the rectangle with the side* a *equal to the side* b*,* $a = b$*, and with diagonal(s)* d *into the diagonal(s)* dn_d *of the rectangle with the sides* an_a*,* bn_b *and with diagonal(s)* dn_d*:*

$$dn_d = ad_1n_{d_1} \tag{23}$$

or

$$dn_d = bd_1n_{d_1} \tag{24}$$

3 Proof of Rectangle Theorem

Proof of Rectangle Theorem. Rectangle MNOP [Fig.2] is a rectangle with the side an_a, bn_b and with diagonal(s) dn_d where assumed that:

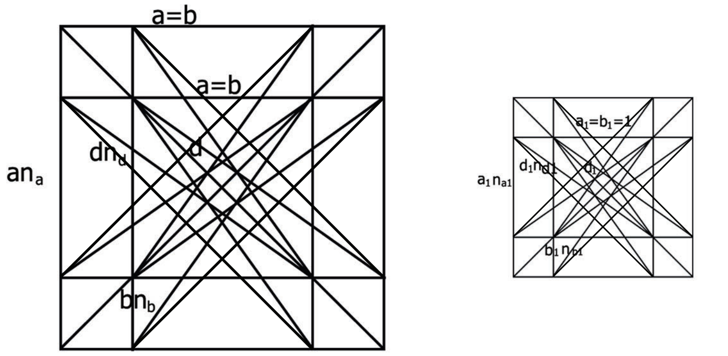

Figure 1: Transformations of a rectangle corresponding to transformations of the rectangle with the side a_1 equal to the side b_1 equal to 1, $a_1 = b_1 = 1$, and with diagonal(s) d_1 equal to $\sqrt{2}$, $d = \sqrt{2}$

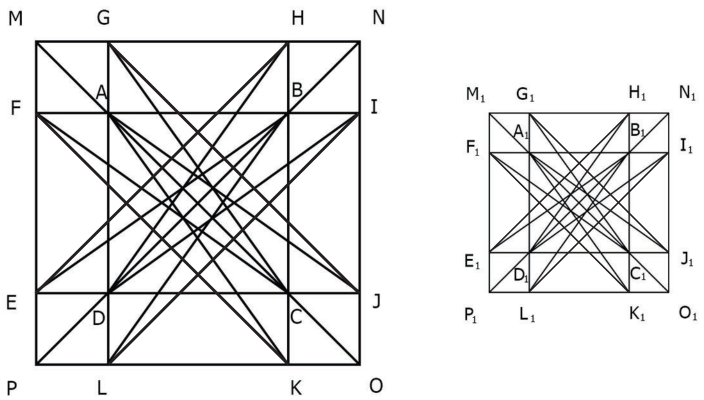

Figure 2: Transformations of the rectangle corresponding to transformations of the rectangle with the side a_1 equal to the side b_1 equal to 1, $a_1 = b_1 = 1$, and with diagonal(s) d_1 equal to $\sqrt{2}$, $d = \sqrt{2}$

$$|\mathrm{MN}| = |\mathrm{PO}| = an_a = 2d - a = 2\sqrt{2}a - a = 2d - b = 2\sqrt{2}b - b$$
$$|\mathrm{MP}| = |\mathrm{NO}| = bn_b = 2d - a = 2\sqrt{2}a - a = 2d - b = 2\sqrt{2}b - b$$
$$|\mathrm{MO}| = |\mathrm{PN}| = dn_d = (2d-a)\sqrt{2} = (2\sqrt{2}a-a)\sqrt{2} = (2d-b)\sqrt{2} = (2\sqrt{2}b-b)\sqrt{2}.$$

Rectangle MNOP [Fig.2] is a transformation of a rectangle ABCD [Fig.2] with a side a equal to a side b, $a = b$, and what proved by Euclid that: "In right-angled triangles, the square on the side subtending the right-angle is equal to the (sum of the) squares on the sides containing the right-angle" [Euclid, Proposition 1.47] with diagonal(s) d equal to $\sqrt{2}a$ equal to $\sqrt{2}b$ into rectangles: ECBF, GDCH, IADJ, KBAL [Fig.2] where assumed that:

$|\mathrm{FE}| = |\mathrm{BC}| = an_a = a * 1 = a = b * 1 = b$
$|\mathrm{FB}| = |\mathrm{EC}| = bn_b = a * \sqrt{2} = \sqrt{2}a = b * \sqrt{2} = \sqrt{2}b$
$|\mathrm{FC}| = |\mathrm{EB}| = dn_d = a * \sqrt{3} = \sqrt{3}a = b * \sqrt{3} = \sqrt{3}b$

$|\mathrm{GH}| = |\mathrm{DC}| = an_a = a * 1 = a = b * 1 = b$
$|\mathrm{GD}| = |\mathrm{HC}| = bn_b = a * \sqrt{2} = \sqrt{2}a = b * \sqrt{2} = \sqrt{2}b$
$|\mathrm{GC}| = |\mathrm{HD}| = dn_d = a * \sqrt{3} = \sqrt{3}a = b * \sqrt{3} = \sqrt{3}b$

$|\mathrm{AD}| = |\mathrm{IJ}| = an_a = a * 1 = a = b * 1 = b$
$|\mathrm{AI}| = |\mathrm{DJ}| = bn_b = a * \sqrt{2} = \sqrt{2}a = b * \sqrt{2} = \sqrt{2}b$
$|\mathrm{AJ}| = |\mathrm{DI}| = dn_d = a * \sqrt{3} = \sqrt{3}a = b * \sqrt{3} = \sqrt{3}b$

$|\mathrm{AB}| = |\mathrm{LK}| = an_a = a * 1 = a = b * 1 = b$
$|\mathrm{AL}| = |\mathrm{BK}| = bn_b = a * \sqrt{2} = \sqrt{2}a = b * \sqrt{2} = \sqrt{2}b$
$|\mathrm{AK}| = |\mathrm{LB}| = dn_d = a * \sqrt{3} = \sqrt{3}a = b * \sqrt{3} = \sqrt{3}b$

by a change of the side b into diagonal d accordingly by:

$n_a = \frac{a}{a} = \frac{b}{b} = 1 = n_{a_1}$
$n_b = \frac{\sqrt{2}a}{a} = \frac{\sqrt{2}b}{b} = \sqrt{2} = n_{b_1}$
$n_d = \frac{\sqrt{3}a}{a} = \frac{\sqrt{3}b}{b} = \sqrt{3} = n_{d_1}$

and into rectangles: MECH, GDJN, LAIO, PFBK [Fig.2] where assumed that:

$|\mathrm{ME}| = |\mathrm{HC}| = an_a = a * \sqrt{2} = \sqrt{2}a = b * \sqrt{2} = \sqrt{2}b$
$|\mathrm{MH}| = |\mathrm{EC}| = bn_b = a * \sqrt{2} = \sqrt{2}a = b * \sqrt{2} = \sqrt{2}b$
$|\mathrm{EH}| = |\mathrm{MC}| = dn_d = a * \sqrt{4} = \sqrt{4}a = b * \sqrt{4} = \sqrt{4}b$

$|\mathrm{GN}| = |\mathrm{DJ}| = an_a = a * \sqrt{2} = \sqrt{2}a = b * \sqrt{2} = \sqrt{2}b$
$|\mathrm{GD}| = |\mathrm{NJ}| = bn_b = a * \sqrt{2} = \sqrt{2}a = b * \sqrt{2} = \sqrt{2}b$
$|\mathrm{DN}| = |\mathrm{GJ}| = dn_d = a * \sqrt{4} = \sqrt{4}a = b * \sqrt{4} = \sqrt{4}b$

$|\mathrm{AL}| = |\mathrm{IO}| = an_a = a * \sqrt{2} = \sqrt{2}a = b * \sqrt{2} = \sqrt{2}b$

$|\text{AI}| = |\text{LO}| = bn_b = a*\sqrt{2} = \sqrt{2}a = b*\sqrt{2} = \sqrt{2}b$
$|\text{AO}| = |\text{IL}| = dn_d = a*\sqrt{4} = \sqrt{4}a = b*\sqrt{4} = \sqrt{4}b$

$|\text{FB}| = |\text{PK}| = an_a = a*\sqrt{2} = \sqrt{2}a = b*\sqrt{2} = \sqrt{2}b$
$|\text{FP}| = |\text{BK}| = bn_b = a*\sqrt{2} = \sqrt{2}a = b*\sqrt{2} = \sqrt{2}b$
$|\text{FK}| = |\text{PB}| = dn_d = a*\sqrt{4} = \sqrt{4}a = b*\sqrt{4} = \sqrt{4}b$

by a change of the side a into diagonal d and the side b into diagonal d accordingly by:
$n_a = \frac{\sqrt{2}a}{a} = \frac{\sqrt{2}b}{b} = \sqrt{2} = n_{a_1}$
$n_b = \frac{\sqrt{2}a}{a} = \frac{\sqrt{2}b}{b} = \sqrt{2} = n_{b_1}$
$n_d = \frac{\sqrt{4}a}{a} = \frac{\sqrt{4}b}{b} = \sqrt{4} = n_{d_1}$

Rectangle $M_1N_1O_1P_1$ [Fig.2] is a rectangle with the side $a_1n_{a_1}$, $b_1n_{b_1}$ and with diagonal(s) $d_1n_{d_1}$ where assumed that:
$|M_1N_1| = |P_1O_1| = a_1n_{a_1} = 2d_1 - a_1 = 2\sqrt{2}a_1 - a_1 = 2d_1 - b_1 = 2\sqrt{2}b_1 - b_1 = 2\sqrt{2} - 1$
$|M_1P_1| = |N_1O_1| = b_1n_{b_1} = 2d_1 - a_1 = 2\sqrt{2}a_1 - a_1 = 2d_1 - b_1 = 2\sqrt{2}b_1 - b_1 = 2\sqrt{2} - 1$
$|M_1O_1| = |P_1N_1| = d_1n_{d_1} = (2d_1 - a_1)\sqrt{2} = (2\sqrt{2}a_1 - a_1)\sqrt{2} = (2d_1 - b_1)\sqrt{2} = (2\sqrt{2}b_1 - b_1)\sqrt{2} = (2\sqrt{2} - 1)\sqrt{2}$.

Rectangle $M_1N_1O_1P_1$ [Fig.2] is a transformation of a rectangle $A_1B_1C_1D_1$ [Fig.2] with a side a_1 equal to a side b_1 equal to 1, $a_1 = b_1 = 1$, and with proved by Euclid that: "In right-angled triangles, the square on the side subtending the right-angle is equal to the (sum of the) squares on the sides containing the right-angle" [Euclid, Proposition 1.47] diagonal(s) d_1 equal to $\sqrt{2}a_1$ equal to $\sqrt{2}b_1$ equal to $\sqrt{2}$, $d_1 = \sqrt{2}a_1 = \sqrt{2}b_1 = \sqrt{2}$, into rectangles: $E_1C_1B_1F_1, G_1D_1C_1H_1, I_1A_1D_1J_1, K_1B_1A_1L_1$ [Fig.2] where assumed that:
$|F_1E_1| = |B_1C_1| = a_1n_{a_1} = a_1 * 1 = a_1 = b_1 * 1 = b_1 = 1$
$|F_1B_1| = |E_1C_1| = b_1n_{b_1} = a_1 * \sqrt{2} = \sqrt{2}a_1 = b_1 * \sqrt{2} = \sqrt{2}b_1 = \sqrt{2}$
$|F_1C_1| = |E_1B_1| = d_1n_{d_1} = a_1 * \sqrt{3} = \sqrt{3}a_1 = b_1 * \sqrt{3} = \sqrt{3}b_1 = \sqrt{3}$

$|G_1H_1| = |D_1C_1| = a_1n_{a_1} = a_1 * 1 = a_1 = b_1 * 1 = b_1 = 1$
$|G_1D_1| = |H_1C_1| = b_1n_{b_1} = a_1 * \sqrt{2} = \sqrt{2}a_1 = b_1 * \sqrt{2} = \sqrt{2}b_1 = \sqrt{2}$

$|G_1C_1| = |H_1D_1| = d_1n_{d_1} = a_1 * \sqrt{3} = \sqrt{3}a_1 = b_1 * \sqrt{3} = \sqrt{3}b_1 = \sqrt{3}$

$|A_1D_1| = |I_1J_1| = a_1n_{a_1} = a_1 * 1 = a_1 = b_1 * 1 = b_1 = 1$
$|A_1I_1| = |D_1J_1| = b_1n_{b_1} = a_1 * \sqrt{2} = \sqrt{2}a_1 = b_1 * \sqrt{2} = \sqrt{2}b_1 = \sqrt{2}$
$|A_1J_1| = |D_1I_1| = d_1n_{d_1} = a_1 * \sqrt{3} = \sqrt{3}a_1 = b_1 * \sqrt{3} = \sqrt{3}b_1 = \sqrt{3}$

$|A_1B_1| = |L_1K_1| = a_1n_{a_1} = a_1 * 1 = a_1 = b_1 * 1 = b_1 = 1$
$|A_1L_1| = |B_1K_1| = b_1n_{b1} = a1 * \sqrt{2} = \sqrt{2}a_1 = b_1 * \sqrt{2} = \sqrt{2}b_1 = \sqrt{2}$
$|A_1K_1| = |L_1B_1| = d_1n_{d_1} = a_1 * \sqrt{3} = \sqrt{3}a_1 = b_1 * \sqrt{3} = \sqrt{3}b_1 = \sqrt{3}$

by a change of the side b_1 into diagonal d_1 accordingly by:
$n_{a_1} = \frac{a_1}{a_1} = \frac{b_1}{b_1} = 1 = n_a$
$n_{b_1} = \frac{\sqrt{2}a_1}{a_1} = \frac{\sqrt{2}b_1}{b_1} = \sqrt{2} = n_b$
$n_{d_1} = \frac{\sqrt{3}a_1}{a_1} = \frac{\sqrt{3}b_1}{b_1} = \sqrt{3} = n_d$

and into rectangles: $M_1E_1C_1H_1, G_1D_1J_1N_1, L_1A_1I_1O_1, P_1F_1B_1K_1$ [Fig.2] where assumed that:
$|M_1E_1| = |H_1C_1| = a_1n_{a_1} = a_1 * \sqrt{2} = \sqrt{2}a_1 = b_1 * \sqrt{2} = \sqrt{2}b_1 = \sqrt{2}$
$|M_1H_1| = |E_1C_1| = b_1n_{b_1} = a_1 * \sqrt{2} = \sqrt{2}a_1 = b_1 * \sqrt{2} = \sqrt{2}b_1 = \sqrt{2}$
$|E_1H_1| = |M_1C_1| = d_1n_{d_1} = a_1 * \sqrt{4} = \sqrt{4}a_1 = b_1 * \sqrt{4} = \sqrt{4}b_1 = \sqrt{4}$

$|G_1N_1| = |D_1J_1| = a_1n_{a_1} = a_1 * \sqrt{2} = \sqrt{2}a_1 = b_1 * \sqrt{2} = \sqrt{2}b = \sqrt{2}$
$|G_1D_1| = |N_1J_1| = b_1n_{b_1} = a_1 * \sqrt{2} = \sqrt{2}a_1 = b_1 * \sqrt{2} = \sqrt{2}b_1 = \sqrt{2}$
$|D_1N_1| = |G_1J_1| = d_1n_{d_1} = a_1 * \sqrt{4} = \sqrt{4}a_1 = b_1 * \sqrt{4} = \sqrt{4}b_1 = \sqrt{4}$

$|A_1L_1| = |I_1O_1| = a_1n_{a_1} = a_1 * \sqrt{2} = \sqrt{2}a_1 = b_1 * \sqrt{2} = \sqrt{2}b_1 = \sqrt{2}$
$|A_1I_1| = |L_1O_1| = b_1n_{b_1} = a_1 * \sqrt{2} = \sqrt{2}a_1 = b_1 * \sqrt{2} = \sqrt{2}b_1 = \sqrt{2}$
$|A_1O_1| = |I_1L_1| = d_1n_{d_1} = a_1 * \sqrt{4} = \sqrt{4}a_1 = b_1 * \sqrt{4} = \sqrt{4}b_1 = \sqrt{4}$

$|F_1B_1| = |P_1K_1| = a_1n_{a_1} = a_1 * \sqrt{2} = \sqrt{2}a_1 = b_1 * \sqrt{2} = \sqrt{2}b_1 = \sqrt{2}$
$|F_1P_1| = |B_1K_1| = b_1n_{b_1} = a_1 * \sqrt{2} = \sqrt{2}a_1 = b_1 * \sqrt{2} = \sqrt{2}b_1 = \sqrt{2}$
$|F_1K_1| = |P_1B_1| = d_1n_{d_1} = a_1 * \sqrt{4} = \sqrt{4}a_1 = b_1 * \sqrt{4} = \sqrt{4}b_1 = \sqrt{4}$

by a change of the side a_1 into diagonal d_1 and the side b_1 into diagonal d_1 accordingly by:
$n_{a_1} = \frac{\sqrt{2}a_1}{a_1} = \frac{\sqrt{2}b_1}{b_1} = \sqrt{2} = n_a$
$n_{b_1} = \frac{\sqrt{2}a_1}{a_1} = \frac{\sqrt{2}b_1}{b_1} = \sqrt{2} = n_b$

$n_{d_1} = \frac{\sqrt{4}a_1}{a_1} = \frac{\sqrt{4}b_1}{b_1} = \sqrt{4} = n_d$

what gives accordingly:

in reference to rectangle MNOP [Fig.2]:
$an_a = aa_1n_{a_1} = ba_1n_{a_1} = a * (2\sqrt{2} - 1) = 2\sqrt{2}a - a = b * (2\sqrt{2} - 1) = 2\sqrt{2}b - b$
$bn_b = ab_1n_{b_1} = bb_1n_{b_1} = a * (2\sqrt{2} - 1) = 2\sqrt{2}a - a = b * (2\sqrt{2} - 1) = 2\sqrt{2}b - b$
$dn_d = ad_1n_{d_1} = bd_1n_{d_1} = a*(2\sqrt{2}-1)*\sqrt{2} = (2\sqrt{2}a-a)\sqrt{2} = b*(2\sqrt{2}-1)*\sqrt{2} = (2\sqrt{2}b - b)\sqrt{2}$

in reference to rectangles ECBF, GDCH, IADJ, KBAL [Fig.2]:
$an_a = aa_1n_{a_1} = ba_1n_{a_1} = a * 1 = a = b * 1 = b$
$bn_b = ab_1n_{b_1} = bb_1n_{b_1} = a * \sqrt{2} = a\sqrt{2} = b * \sqrt{2} = b\sqrt{2}$
$dn_d = ad_1n_{d_1} = bd_1n_{d_1} = a * \sqrt{3} = a\sqrt{3} = b * \sqrt{3} = b\sqrt{3}$

in reference to rectangles MECH, GDJN, LAIO, PFBK [Fig.2]:
$an_a = aa_1n_{a_1} = ba_1n_{a_1} = a * \sqrt{2} = a\sqrt{2} = b * \sqrt{2} = b\sqrt{2}$
$bn_b = ab_1n_{b_1} = bb_1n_{b_1} = a * \sqrt{2} = a\sqrt{2} = b * \sqrt{2} = b\sqrt{2}$
$dn_d = ad_1n_{d_1} = bd_1n_{d_1} = a * \sqrt{4} = a\sqrt{4} = b * \sqrt{4} = b\sqrt{4}$

According to proved by Euclid that: "In right-angled triangles, the square on the side subtending the right-angle is equal to the (sum of the) squares on the sides containing the right-angle" [Euclid, Proposition 1.47] in rectangles ECBF, GDCH, IADJ, KBAL [Fig.2] where assumed that: $an_a = a = b$, $bn_b = \sqrt{a^2 + a^2} = \sqrt{2}a = \sqrt{b^2 + b^2} = \sqrt{2}b$ diagonal(s) dn_d is(are) equal to the square root of the sum of the squares of an_a and bn_b, $dn_d = \sqrt{a^2 + (\sqrt{2}a)^2} = \sqrt{3}a = \sqrt{b^2 + (\sqrt{2}b)^2} = \sqrt{3}b$ and in rectangles MECH, GDJN, LAIO, PFBK [Fig.2] where assumed that: $an_a = \sqrt{a^2 + a^2} = \sqrt{2}a = \sqrt{b^2 + b^2} = \sqrt{2}b$, $bn_b = \sqrt{a^2 + a^2} = \sqrt{2}a = \sqrt{b^2 + b^2} = \sqrt{2}b$ diagonal(s) dn_d is(are) equal to the square root of the sum of the squares of an_a and bn_b, $dn_d = \sqrt{(\sqrt{2}a)^2 + (\sqrt{2}a)^2} = \sqrt{4}a = \sqrt{(\sqrt{2}b)^2 + (\sqrt{2}b)^2} = \sqrt{4}b$.

Rectangle MNOP [Fig.2] is a square on side an_a or bn_b, $MNOP = (an_a)^2 =$

$(bn_b)^2$, [Euclid, Proposition 1.46] equal to the sum of areas of rectangles ECBF, GDCH, IADJ, KBAL [Fig.2] and MECH, GDCH, LAIO, PFBK [Fig.2] reduced by multiplied areas accordingly, $MNOP = (an_a)^2 = (bn_b)^2 = 2(\sqrt{2}a)^2 - a^2 + 2(\sqrt{2}a)^2 - 4a(\sqrt{2}a) + 2a^2 = 2(\sqrt{2}b)^2 - b^2 + 2(\sqrt{2}b)^2 - 4b(\sqrt{2}b) + 2b^2 = (2\sqrt{2}a - a)^2$

$an_a = 2\sqrt{2}a - a = aa_1n_{a_1} = ba_1n_{a_1}$

$bn_b = 2\sqrt{2}b - b = ab_1n_{b_1} = bb_1n_{b_1}$

$dn_d = (2\sqrt{2}a - a)\sqrt{2} = (2\sqrt{2}b - b)\sqrt{2} = ad_1n_{d_1} = bd_1n_{d_1}$. □

References

[Euclid] Euclid of Alexandria, *Euclid's Elements of Geometry, edited, and provided with a modern English translation, by Richard Fitzpatrick, ISBN 978-0-6151-7984-1*

List of Figures

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